The Best Linear Approximation of MIMO systems: simplified nonlinearity assessment using a toolbox

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Abstract

The aim of the paper is to introduce a case study for nonparametric nonlinear assessment using a toolbox illustrated on a small, battery-operated road vehicle. The toolbox (Simplified Analysis for Multiple Input Systems, SAMI) is based on the Best Linear Approximation framework of multiple input, multiple output systems which provides a user-friendly interpretation of the measurement data by extracting the user relevant information by splitting up the classical coherence function of the FRF measurements into noise, nonlinearity and transient parts. By the use of the proposed toolbox, a less experienced user will be able to 1) nonparametrically assess the system under test, 2) to tell whether the underlying system is linear or not, and 3) to tell how much is the gain by the use of an advanced nonlinear framework. The proposed toolbox addresses the questions related to the semi-automatic processing of MIMO measurements with respect to the whole process from the design of experiment to the nonparametric analysis of the frequency response functions. The toolbox has been tested with simulations and real-life industrial measurements.

1 Introduction

The goal of this paper is to introduce a simple but efficient state-of-the-art nonparametric methodology for modeling and analyzing vibrating structures in real-life experimental situations while providing practical advice to the inexperienced users. The proposed methodology is illustrated on – but not limited to – a battery-operated small road vehicle called Simrod.

Many mechanical and civil structures are inherently nonlinear. The problem lies in the fact that there is no unique modelling tool because there are many different types of nonlinear systems – each of them behaves differently –therefore modelling is very involved, and universally usable design and modelling tools are not available. For these reasons the nonlinear systems are often approximated with linear systems.

This paper introduces the Best Linear Approximation (BLA) framework for measurements of MIMO (multiple input, multiple output) systems which provides a user-friendly interpretation of the measurement data by extracting the user relevant information. The key idea in the proposed approach is to make use of some statistical properties of the excitation signal such that is becomes possible to split up the classical coherence function of the FRF measurement into noise and nonlinearity information. The (non)linearity assessment is based on the interpretation of the FRF, noise and nonlinearity estimates.

For MIMO systems, the design of experiment is more involved than in the single input case, and the necessary time which is needed to analyze the available FRFs grows with the number of input and output channels. Further, to properly analyze the estimation results and to troubleshoot, an experienced user is needed. To overcome the above-mentioned issues, a toolbox is proposed which addresses the questions related to the semi-automatic processing of MIMO measurements with respect to the whole nonlinearity assessment process.

The proposed estimation toolbox (Simplified Analysis for Multiple Input Systems (SAMI) is a result of an industrial research program. The work has been tested with simulations and real-life industrial measurements. For a detailed overview of the components of the framework we refer to [1], for an overview of the toolbox functionalities we refer to [2].

This paper is organized as follows. Section 2 elaborates the considered systems and the main assumptions applied in this work. Section 3 briefly describes the toolbox. Section 4 addresses questions related to the experiment design. Section 5 discusses the estimation framework in detail. In Section 6 the description and analysis of the vibration testing of a small battery-operated vehicle. Conclusions can be found in Section 7.

2 Basics

2.1 Definitions and assumptions

Several definitions and assumptions must be addressed prior to carrying out any system identification procedure. The considered systems are mechanical or civil dynamic vibrating structures. The dynamics of a linear MIMO system can be nonparametrically characterized in the frequency domain by its Frequency Response Matrix (FRM, a matrix whose elements are FRFs [3]) G at frequency index k, which relates n_i inputs U to n_o outputs Y of N measurement samples as follows:

$$Y[k] = G[k]U[k] \tag{1}$$

where $G[k] \in \mathbb{C}^{n_o \times n_i}$, $Y[k] \in \mathbb{C}^{n_o \times 1}, U[k] \in \mathbb{C}^{n_i \times 1}$, $k = 0 \ldots \frac{N-1}{2}$ at frequency $f_k = \frac{k f_s}{N}$ with sampling frequency of f_s. In order to make the text more accessible, the frequency indices and dimensionalities will be omitted.

The system represented by G is linear when the superposition principle is satisfied in steady state, i.e.:

$$Y = G\{(a+b)U\} = a\, G\{U\} + b\, G\{U\} = (a+b)\, G\{U\} \tag{2}$$

where a and b are scalar values. If G is constant, for any a, b (and excitation), then the system is called linear. On the other hand, when G varies with a and b (and the variation depends also on the excitation signal – e.g. level of excitation, distribution, etc.) then the system is called nonlinear.

In this work an arbitrary number of input and output channels are considered. The underlying systems are damped, bounded-input, bounded-output stable, time-invariant, nonlinear systems where the linear response of the system is still present and the output of the underlying system has the same period as the excitation signal (i.e. the system has PISPO behaviour: period in, same period out [3] [4]).

Further, it is assumed that the output Y is measured with additive, independent and identically distributed Gaussian noise with zero mean and finite variance σ_y (denoted as E), and it can contain transient response (denoted as T) such that the measurement $Y_{measured}$ is given by:

$$Y_{measured} = Y + E + T = GU + E + T \tag{3}$$

2.2 Measurement setup and instrumentation

The proposed framework makes use of the classical instrumentation and measurement setups [5]. A general overview of the measurement setup is shown in Figure 1 with an illustration of a car frame testing. The reference signals are the ideal signals generated by the toolbox to excite the system to be tested. In this work these signals are so called multisine (pseudo-random noise) signals (see Section 4.). In the measurement example, the reference signals are the voltage signals being fed to the shakers via power amplifiers which represent the excitation system. The shakers start to move and generate forces which are the input signals to the car frame (i.e. the system under test). The output signals contain the waveforms of the system's reaction to the input signals, and they are measured with a sensor. In this illustration the output signals are the acceleration signals. In this work it is assumed that the SNR of the input signal with respect to one measurement period (block) is at least 20 dB. Note, that most of the mechanical and electrical measurements have a much better SNR, typically well above 40 dB. Further, from instrumentation point of view, it is assumed that the measurement is perfectly synchronized i.e. samples are acquired at the same time instant with constant sampling frequency.

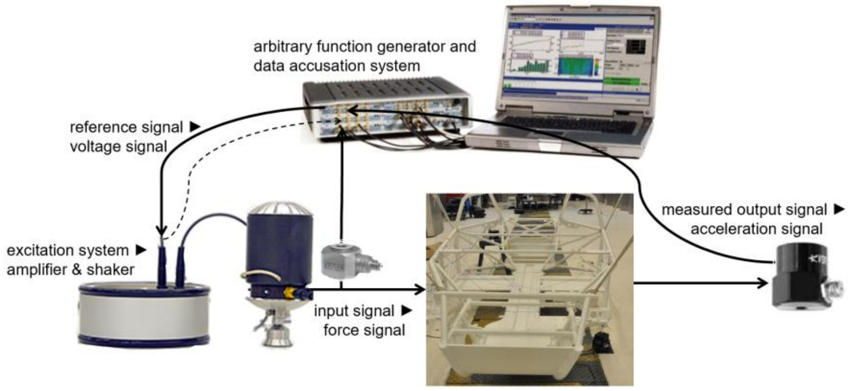

Figure 1. An illustration of the measurement-instrumentation setup used for vibration testing of the Siemens Simrod vehicle. The reference signals (voltage) are generated by the toolbox and they are fed to the excitation system (shaker) which generates the input signals (force).

3 The toolbox

The Simplified Analysis for Multiple Input Systems (SAMI) is a user-friendly Matlab based toolbox [1] [6]. It supports command line and graphical user interfaces. The toolbox has been tested with many simulations and real-life industrial measurements, and it is optimized for MIMO experiments. SAMI supports data import from various platforms, including Simcenter Testlab [7]. The toolbox provides a user-friendly interpretation of the nonlinear MIMO measurement data by extracting the user relevant information and it addresses the questions related to the automatic processing of MIMO measurements with respect to the whole process. The key idea is to make use of the statistical properties of the proposed excitation signal such that it becomes possible to split up the classical coherence function into noise, nonlinearity (and transient) components. The warning-notification system is developed by experiences learned from typical mistakes which popped up during the measurement campaigns. The internal software architecture consists of the following interconnected layers (see Figure 2):

- Step 1: The *Design of experiment* and *measurement* blocks address the issues related to signal design, choice of measurement parameters, instrumentation and execution of the measurement.
- Step 2: The *Pre-processing* block considers a check-up of the input (reference) channels and provides an early warning to the user when the inputs are strongly correlated. Furthermore, it segments the measurement data and sets up the processing parameters for the BLA transfer function estimation.
- Step 3: The *BLA estimation* block provides the BLA FRF estimation, calculates advanced statistics.
- Step 4: The *Post-processing* block provides users with the FRF, noise, nonlinearity, transient estimates and warnings. It is possible to automatically highlight the FRF (or input, output, reference) channels that have significant nonlinearity or noise levels. Furthermore, channels with sensory faults and/or imperfections, and correlated inputs are detected as well.

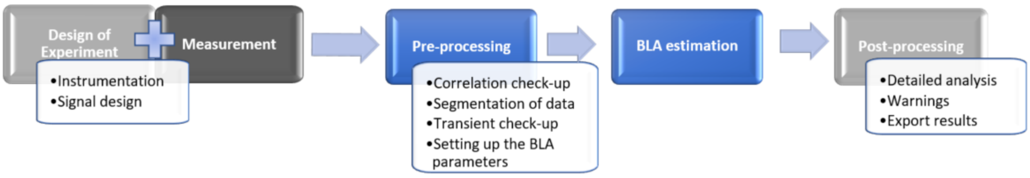

Figure 2: Overview of the proposed estimation framework. Blue blocks require – in normal circumstances – no user interactions. Light gray blocks require moderate user interaction. Dark gray block requires experienced user interaction.

4 Design of experiment

4.1 Multisine excitation

In modern system identification special excitation signals are available to assess the underlying systems in a user-friendly, time efficient way [8]. To avoid any spectral leakage, to reach full nonparametric characterization of the noise, and to be able to detect nonlinearities, a periodic signal is needed. The best signal that satisfies the desired properties is the easy to implement and apply (random phase) multisine signal (see Figure 3) which looks like white noise, behaves like it but it is not a noise. If the multisine contains all/only odd or even harmonics, then it is called full (band)/odd or even multisine.

In general, when a small excitation level is used, the effects of nonlinearities can be hidden in the noise. If the excitation level is higher, the effect of the nonlinearities becomes visible. If a full-band multisine excitation is used, then the details of the nonlinear behavior are not directly separable from the linear part. Figure 4 shows an example, how the system response consists of the linear and nonlinear part. When an excitation set of only even harmonics is used, the odd nonlinearities are not detectable. The solution to detect both even and odd nonlinear contributions is to use an excitation set only with odd harmonics, e.g. odd random phase multisine. In order to examine the odd frequencies, it is needed to skip several odd harmonics in the multisine. The experiences show that the odd, random phase multisine with randomly skipped harmonics is the best what can be used because when a fixed odd harmonic is missing then certain type of nonlinearities can be hidden. A random skipping is recommended to be done within a fixed group of harmonics. Note that this signal is also known as a pseudo-random noise signal. For a detailed overview of excitation signals we refer to [3] [5].

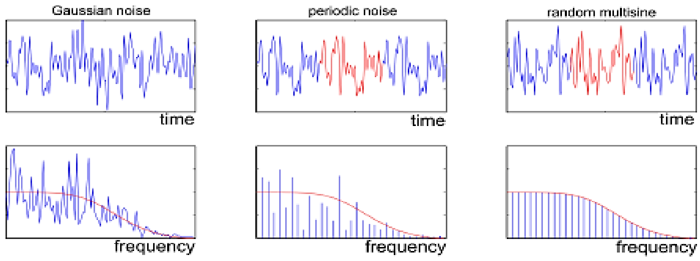

Figure 3: Different excitation signals in time and in frequency domain

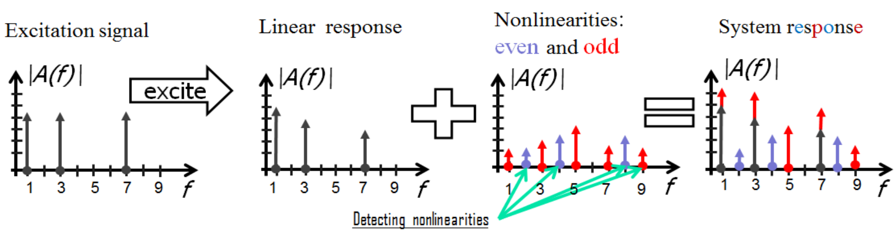

Figure 4: System response originating from linear and nonlinear part of excited system

4.2 Multisines for multiple input measurements

Without loss of generality, we will focus on systems with two inputs and two outputs. The straightforward extension of the SISO excitation case can be formulated in the frequency domain as follows:

$$Y = GU \Leftrightarrow \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \qquad (4)$$

where the indices of the input and output data refer to the channel number.

The above-mentioned set of linear equations suffers from the degrees of freedom problem: there are four unknown parameters and only two independent equations. In order to be able to solve the above-mentioned algebraic equation, the number of independent equations has to be increased. This can be done by increasing the number of experiments: in case of 2 inputs there are at least 2 different experiments needed.

We recommend using so-called orthogonal random multisines in order to achieve uncorrelated experiments [1, 9]. Orthogonal multisines are based on the concept of the orthogonal inputs for linear MIMO measurements introduced in [10]. The proposed procedure is to generate independent random excitations for every input channel such that we have (more) randomness in the measurement with respect to the classical Hadamard's technique [11] [12]. For each input channel there is an independently generated multisine sequence assigned.

It is crucial to highlight that it is possible to use multiple periods (blocks) of multisines. Using multiple periods, the SNR of the measurement will be improved. Furthermore, it is highly recommended to enrich the randomness of the (periodic) multisines by adding multiple random realizations. Increasing the number of random realizations results in more robust nonlinearity estimates. In case of M independent random realizations and P repeated periods, the excitation signal is given by:

$$U = \overbrace{\begin{bmatrix} \overbrace{U_{11}^{(1)} \dots U_{11}^{(1)}}^{P-times} & \overbrace{U_{12}^{(1)} \dots U_{12}^{(1)}}^{P-times} \\ \overbrace{U_{21}^{(1)} \dots U_{21}^{(1)}}^{P-times} & \overbrace{U_{22}^{(1)} \dots U_{22}^{(1)}}^{P-times} \end{bmatrix}}^{1^{st}\ set\ of\ indepedent\ U\ signals} \dots \overbrace{\begin{bmatrix} \overbrace{U_{11}^{(M)} \dots U_{11}^{(M)}}^{P-times} & \overbrace{U_{12}^{(M)} \dots U_{12}^{(M)}}^{P-times} \\ \overbrace{U_{21}^{(M)} \dots U_{21}^{(M)}}^{P-times} & \overbrace{U_{22}^{(M)} \dots U_{22}^{(M)}}^{P-times} \end{bmatrix}}^{M^{th}\ set\ of\ indepedent\ U\ signals} \tag{5}$$

5 Best Linear Approximation framework

The Best Linear Approximation (BLA) of a nonlinear system is a modelling approach that minimizes the mean square error between the measured output of a nonlinear system and the output of the linear model [8]. It makes use of the knowledge that the excitation signal has both stochastic and deterministic properties. The excitation signal is a random phase multisine signal with M different (random phase) realizations, each of the blocks (periods) is repeated P times. By using multiple realizations (i.e. random phase rotations) of the multisines, the richness of the signal is increased. Instead of directly using the averaged input and output data (i.e. the use of classical H1 framework [5] [13]), a partial BLA estimate is calculated for each period of the excitation. A BLA FRM estimate, for a given signal, is then calculated via the average of partial BLA estimates (see Figure 2). In this case we can easily estimate the noise and nonlinearity levels. Figure 5 shows the theoretical structure of the considered BLA estimator.

G_{linear} is the linearized (transfer function), phase coherent component of the model for which phase rotations at the input result in a proportional phase rotation at the output [4]. For the cases, when the output has phase non-coherent behavior, we can capture the (non-coherent) nonlinearities represented by G_S. Increasing the number of random realizations of the multisine signal allows us to tackle the random output phase rotations as a 'half-stochastic' (nonlinear) noise source. The 'half-stochastic' term refers to the fact that G_S does not vary over the repetitions of the same signal segment but only over different realizations. G_E represents the classical FRF measurement noise. The usage of periodic excitation reduces the effects of the measurement noise component G_E (see (10)). The usage of multiple realizations reduces the impact of non-coherent nonlinearities represented by G_S. G_{Bias} represents the nonlinearities remaining after multiple realizations of the excitation signal.

The considered steady-state model at period p and realization m at excited frequency bins is given as a straightforward extension of Figure 5 by:

$$\hat{G}^{[m]} = \underbrace{\frac{1}{P}\sum_{p=1}^{P} \overbrace{Y_{measured}^{[m][p]} U^{[m]^{-1}}}^{measured\ FRFs\ in\ m}}_{the\ method\ for\ calculation} = \underbrace{\frac{1}{P}\sum_{p=1}^{P} \overbrace{\hat{G}_{BLA}^{[m]}}^{\substack{FRF\\ estimate}} + \overbrace{\hat{G}_{S}^{[m]}}^{\substack{nonlinearity\\ fixed\ in\ m}} + \overbrace{\hat{G}_{E}^{[m][p]}}^{\substack{noise\\ varies\ over\ p}}}_{theoretical\ values} \tag{6}$$

If there is a transient term present in the measurement it is either discarded or estimated [14]. For computational optimality, the FRM estimation is solved output channel-wise (i.e. one output channel per time). The BLA estimate is calculated at the excited frequency lines only. In the frequency band of excitation, the BLA estimates at the non-excited bins are obtained by (linear) interpolation.

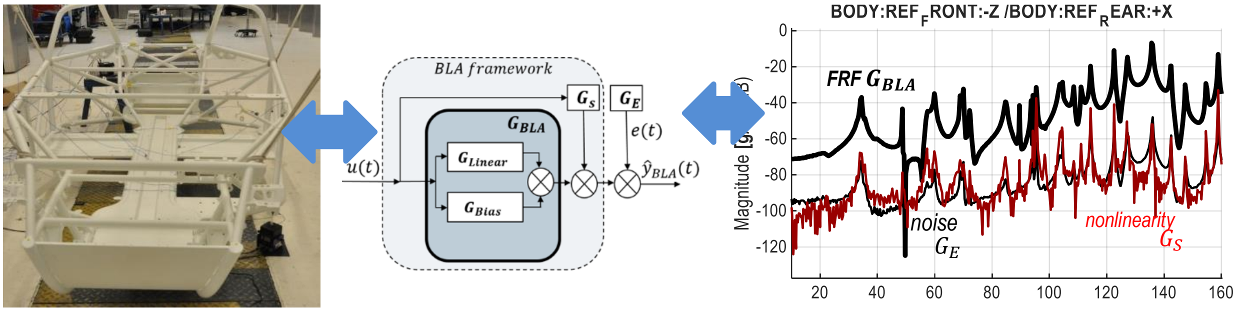

Figure 5: The structure of the best linear approximation (BLA, middle figure) and its connection to the FRF estimate (right figure) illustrated on a vibration testing measurement of an Simrod vehicle (left figure). G_{BLA} refers to the BLA FRF, G_E refers to the noise, G_s refers to nonlinear distortions.

A partial FRM estimate is obtained via averaging over the P periods (blocks) in a realization, see (10). If P is sufficiently large, then (considering the law of large numbers and the distribution properties of the measurement noise) the term $\hat{G}_E^{[m]}$ in (10) converges to zero. Because the stochastic nonlinear contribution $\hat{G}_S^{[m]}$ does not vary over the P repetitions of the same realization, one has to average over the M different realizations. If M is sufficiently large, then the averaged nonlinear noise source $\hat{G}_s$ in (10) converges to zero. After the 2D averaging (i.e. averaging over P and M) the BLA estimate is obtained, see Figure 6.

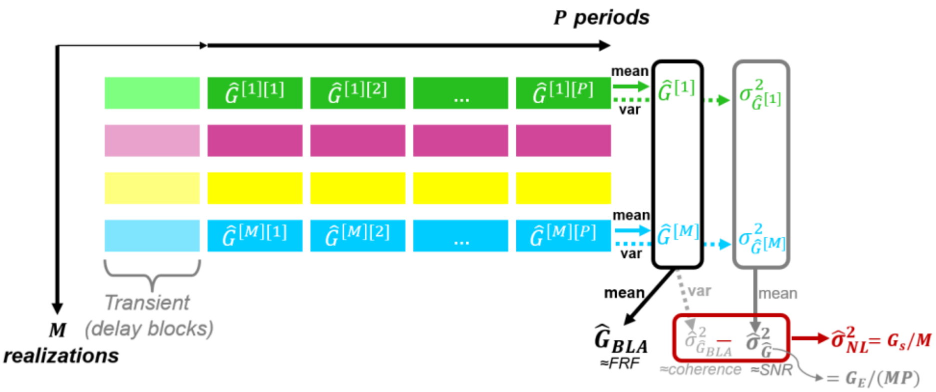

Figure 6: Evaluation of BLA estimate with the help of 2D averaging. A partial BLA estimate $\hat{G}^{[m]}$ and its improved noise estimate $\sigma^2_{\hat{G}[m]}$ are obtained via the period-wise estimates $\hat{G}^{[m][p]}$. The BLA estimate $\hat{G}_{BLA}$ and its improved variance estimates $\hat{\sigma}^2_{\hat{G}_{BLA}}$ are obtained via partial BLA estimates. $\hat{\sigma}^2_{NL}$ and $\hat{\sigma}^2_{\hat{G}}$ stand for the improved (experiment-wise) stochastic nonlinearity and noise estimates. G_S and G_E stand for the period-wise stochastic nonlinearity and noise estimates (see Figure 5).

The noise covariance $\hat{\sigma}^2_{\hat{G}}$ is estimated from the improved averaged sample variance $\hat{\sigma}^2_{\hat{G}[m]}$ of each FRM realization as $\hat{\sigma}^2_{\hat{G}} = \frac{1}{M^2}\sum_{\mathrm{m}=1}^{M} \sigma^2_{\hat{G}[m]}$ with $\hat{\sigma}^2_{\hat{G}[m]} = \sum_{p=1}^{P} \left|\hat{G}^{[m][p]} - \hat{G}^{[m]}\right|^2 / (P(P-1))$ where in $\hat{\sigma}^2_{\hat{G}[m]}$ the additional normalization with P is needed to show the improved covariance (noise) estimate (this term corresponds to G_E/P). In other words, averaging over repeated blocks results in an improvement of the SNR. Similarly, the additional normalization with M in $\hat{\sigma}^2_{\hat{G}}$ is needed to show the noise estimate improvement over different realizations (this term corresponds to G_E/MP). If the user wants to see the covariance (noise) with respect to one period (block) one has to multiply $\hat{\sigma}^2_{\hat{G}}$ with MP (this normalization is used in Figure 5).

The total variance of the FRM $\hat{\sigma}^2_{\hat{G}_{BLA}}$ is estimated from the improved variance (i.e. with extra normalization factor M) of each partial BLA estimate $\hat{G}^{[m]}$ as $\hat{\sigma}^2_{\hat{G}_{BLA}} = \sum_{m=1}^{M} \left|\hat{G}^{[m]} - \hat{G}_{BLA}\right|^2 / (M(M-1))$.

The difference between the total variance and the noise variance is an estimate of the variance of the stochastic nonlinear contributions such that $\hat{\sigma}^2_{NL} = \hat{\sigma}^2_{\hat{G}_{BLA}} - \hat{\sigma}^2_{\hat{G}}$. If the user wants to see nonlinear contributions with respect to one period (block) one has to multiply $\hat{\sigma}^2_{NL}$ with M (this normalization mode is used in Figure 5). For the interpretation of different normalization modes, see Section 6.5.2 For the detailed calculation we refer to [2].

6 Experimental illustration

6.1 The measurement setup

This section concerns a vibration testing measurement campaign of a small battery-operated two persons vehicle called Simrod. A short description of the vehicle and its modal testing can be found at [15]. The car frame is excited by two shakers using combined multisines: odd multisines with skipping one random bin within each group of 4 successively excited odd lines. This sparse grid is used to detect even and odd nonlinear contributions. The sampling frequency is 400 Hz, the period length 4096 samples. The inputs (force) and outputs (acceleration) signals are measured. The range of excitation is between 9.9609 and 159.961 Hz, there are 5 periods and 10 realizations per excitation level. There are 2 different input levels measured at low level in horizontal direction as 2.95, in vertical direction 2.2 N RMS, and in high level 5.94 and 4.95 N RMS, respectively. In the 30 – 150 Hz band, the vehicle frame possesses about 20 resonance modes. In this paper we focus only on the nonparametric analysis of the vehicle.

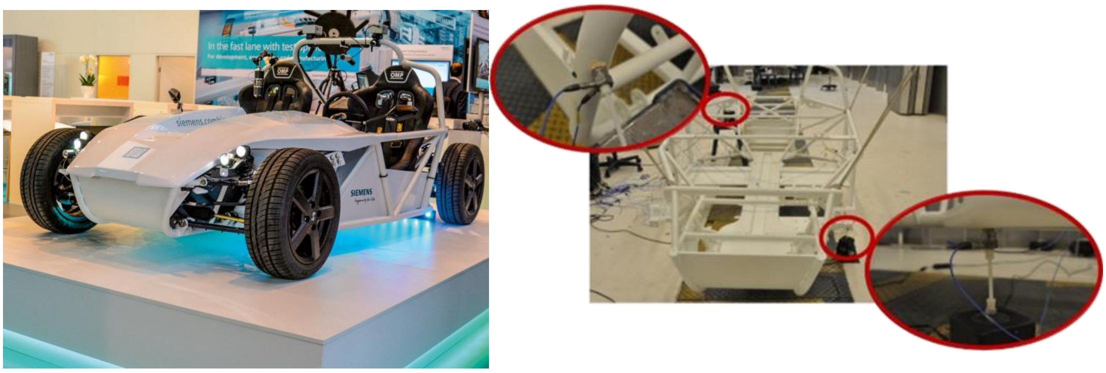

Figure 7: Left: the vehicle is shown in a fully assembled state. Right: the suspended frame of the vehicle is shown together with the excitation shakers.

6.2 Data processing

The data processing is fully automated by the toolbox. In short, the first step is the automated segmentation of data: retrieving the periods and realizations. Next, the trends (such as the mean/offset values) from the individual segments are removed [8], and the transient is analyzed. Last, an early quality assessment of experimental data is done, and the parameters for the estimation procedure are set. Based on the SNRs of the input-output data, existence of reference signal, number of periods and the length of the transient, an appropriate estimator is automatically set. Currently, the automated possibilities include BLA, H1, indirect BLA [8], and special implementations of the LPM (Local Polynomial Method), LRM (Local Rational Method), indirect LPM and LRM methods, see [2] [16]. This section contains the results of the following one-line Matlab code directly used on the data: *CreateAnalyzePlotAIO(Force,Acceleration,[],Fs).*

As can be seen, the toolbox requires only the signals in vector/Matrix form, and optionally the sampling frequency (if it is not set, then a normalized frequency scale is used). The figures shown in this paper are obtained from the toolbox (for accessibility resized, with no warnings).

Figure 8 shows the visualization of toolbox transient check-up routine. In order to determine the length of the transient (i.e. the number of delay blocks), the last block (period) – assumed to be nearly in steady-state – is subtracted from every preceding block. Because the transient decays as an exponential function, the differences are shown in logarithmic scale. Using automated statistical analysis of all available signals, the transient term is estimated as 2 blocks. Because there are more than 2 transient-free periods available, the input-output signals are measured sufficiently good (see next subsection). Because the reference signal is not available, the BLA estimator is automatically selected.

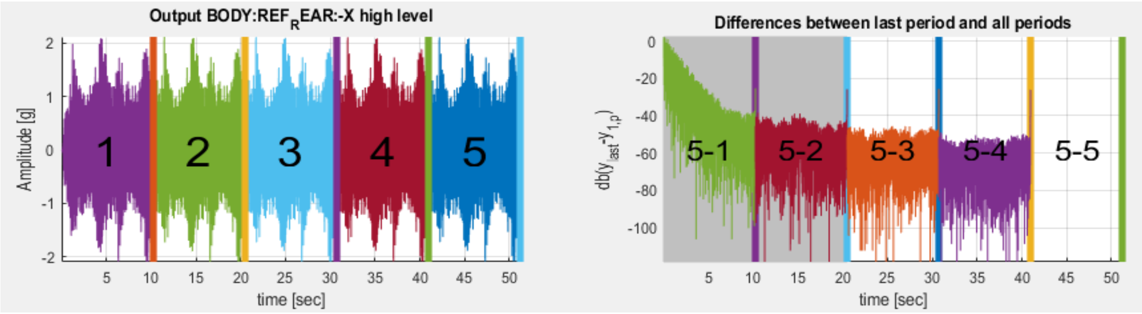

Figure 8: The left bottom figure shows the acceleration measurements at the horizontal driving point. The second figure shows the difference between the last block minus every block. The gray area refers to the automatically detected transient (delay) blocks which will be discarded during the data processing.

6.3 Input signal

It is crucial to analyze the excitation system as well to quantify its accuracy and linearity. For that reason, the input signals (generated multisine signals) are measured as well, see Figure 9. The figure shows the measured signals and their noise estimates. This measurement has good quality (SNR is greater than 40 dB), in the horizontal excitation, in the 10 – 60 Hz band the nonlinear distortions are higher than the noise level. It is interesting to point out that the high input signal is 6,52 dB higher than the low one, but the SNR is only increased around with around 1,33 dB. This indicates the presence of (weak) nonlinearities at the excitation system. Similarly, this information can be seen by looking at the even and odd distortions: the higher the excitation, the higher the nonlinearities. In the vertical excitation, even and odd nonlinear distortions are hidden in the noise.

Overall, the measurement quality (the gap between the actuator transfer function and its nonlinear estimates) is good, much higher than 20 dB needed to fulfil the assumption on the precise excitation signal measurement.

6.4 Output measurement

Further, in order to simplify the analysis, the output and FRF are shown at the driving points only. The output (acceleration) measurements are shown in Figure 10. As can be seen, the SNR is around 40…50 dB at the dominant resonances. Similarly to the input measurement, with increasing the excitation level the noise estimate has moved as well. It can also be observed that the resonances have been shifted a little, which is also a further indication of nonlinearities. This is due to the fact, that at higher level of excitation we have dominant odd distortions, which usually manifest in changing resonance locations and shapes (it is the so-called hardening or softening stiffness nonlinearity effect). Note that even distortions usually manifest as (excessive) noise on the measurement which explains the moving noise level estimates.

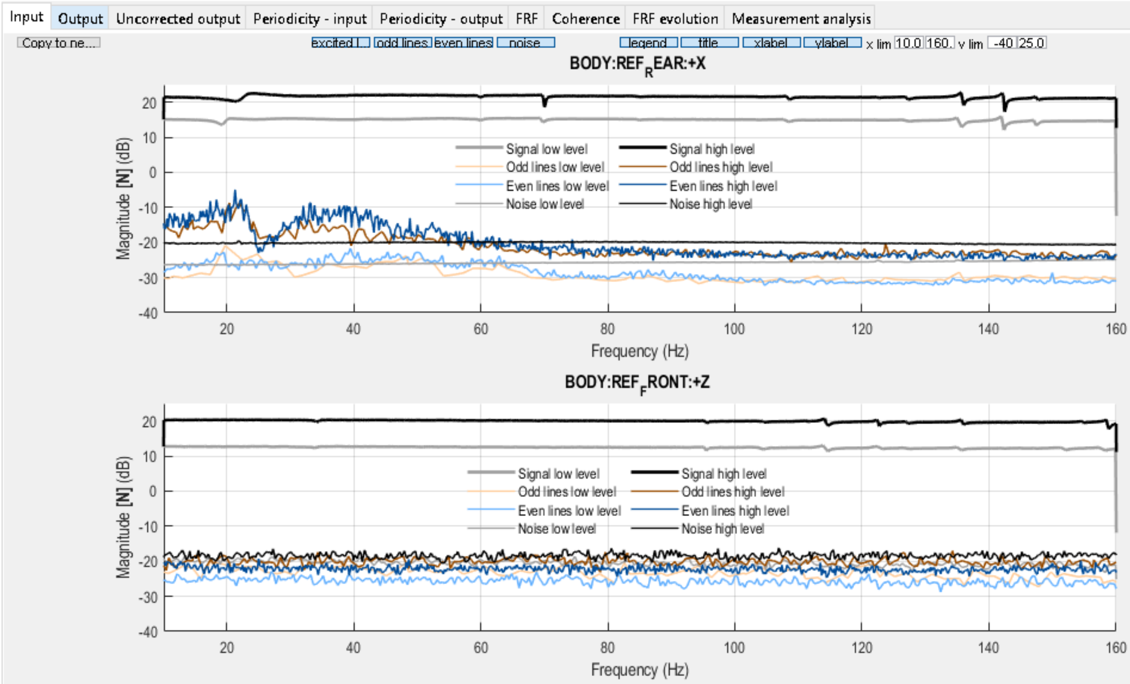

Figure 9: The measured input (force) signals are shown. Darker shades refer to high excitation level. Thick grey shades refer to the signals. Thin grey shades refer to noise estimates. Orange shades refer to the odd distortions. Blue shades refer to the even distortions. Observe that the higher the excitation, the more the nonlinear distortion in the *x* direction.

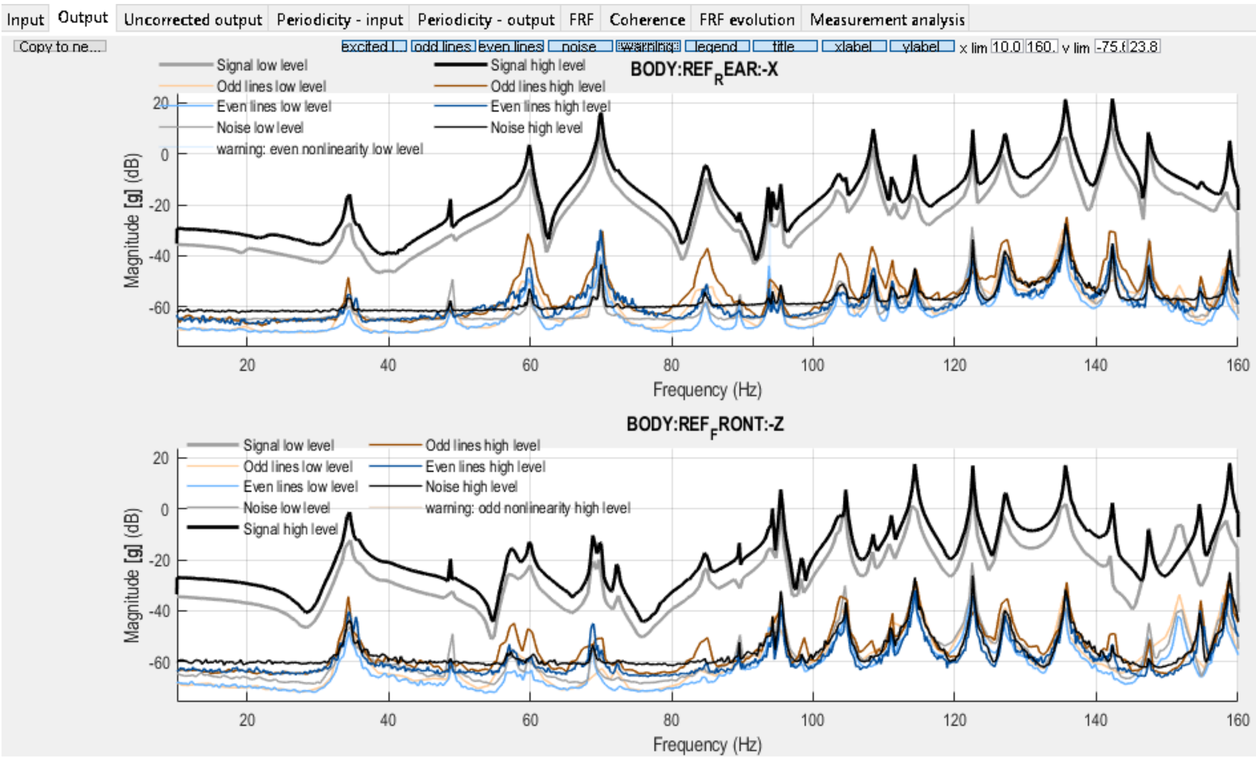

Figure 10: The output (acceleration) measurement shown at the driving points. Darker shades refer to high excitation level. Thick grey shades refer to the signals. Thin grey shades refer to noise estimates. Orange shades refer to the odd distortions. Blue shades refer to the even distortions.

6.5 FRF analysis

6.5.1 FRFs at different excitation levels

Figure 11 shows the FRFs at low and high level of excitation. This is the classical approach, FRFs at multiple level of excitation are compared with each other. It is interesting to point out that despite the fact that the high excitation is only 6.52 dB higher than the low level excitation, it can be observed that FRFs at different levels differ at a certain region (see the encircled regions in Figure 11) from each other. This indicates the presence of (weak) nonlinearities. However, for just one level of excitation, it is not possible to determine if there are nonlinearities present using the classical approach. The usage of the proposed multisines allows us to obtain noise and nonlinearity level estimations as explained in Section 5. With the help of these curves one can distinguish between the effects of noise and nonlinearity.

For instance, when looking at the third FRF (second row, first column, Figure 11) the resonance around 95 Hz (see the dashed encircled area) has an approximate SNR of 34 dB, and an SNLR (signal-to-nonlinearity ratio) of 3 dB. This means that at that resonance the main error source is the nonlinearity. If a linear model is used, then the expected error level will be in the order of the SNLR. If an appropriate nonlinear model is used, the expected error level could drop to the SNR. This kind of extra information would have been impossible to derive from the H1 framework.

Furthermore, the toolbox provides an automated warning system for the FRF, actuator, reference, input, output signals when significant nonlinearities, noise, sensory fault, correlation issues are discovered. This warning system is partly illustrated in Figure 11.

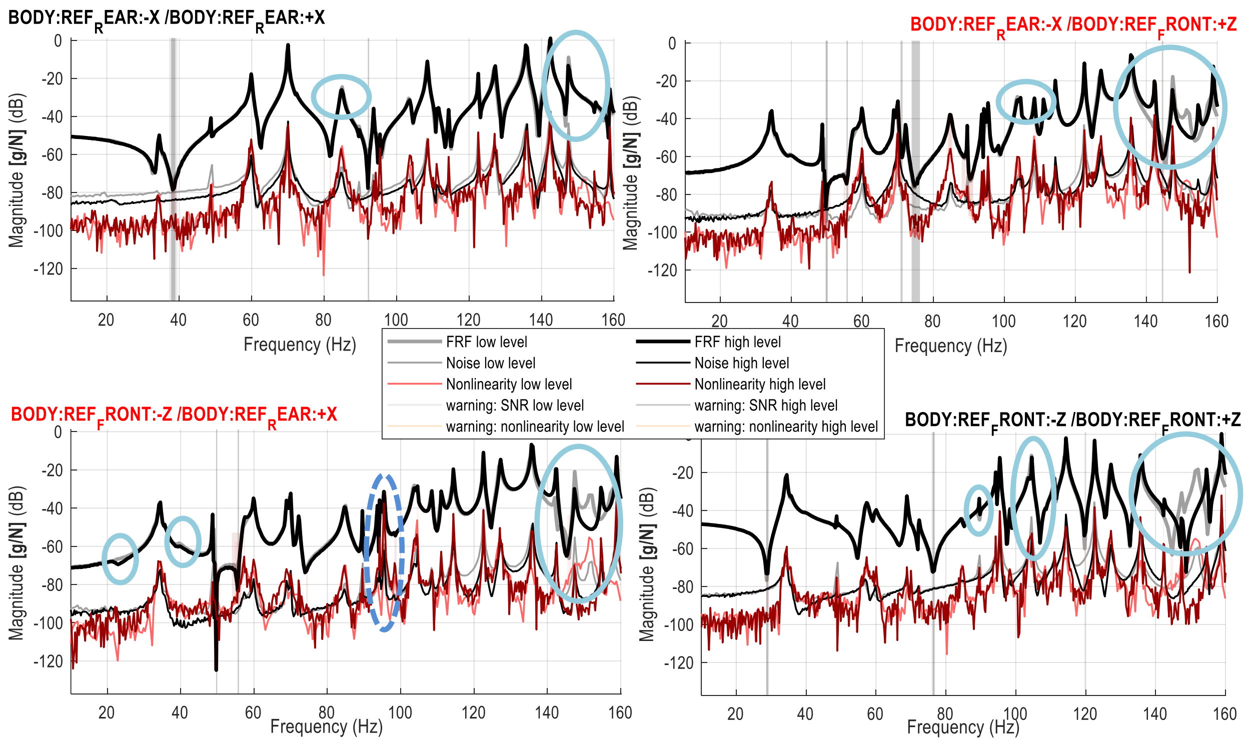

Figure 11: The FRF estimation shown at the driving points. Darker shades refer to higher excitation levels. Thick grey shades refer to the FRFs. Thin grey shades refer to noise estimates. Red shades refer to the nonlinear distortions. The encircled areas refer to the visible changes of the FRFs between high and low level of excitation.

6.5.2 Normalization modes

Figures 9 – 11 show the noise and nonlinearity estimates normalized with respect to one block (period). This normalization mode is very important for understanding, modeling, simulation and control. Of course, it is possible to show the improved noise and nonlinearity estimates with respect to the whole experiment or for one realization of the multisine signals. The toolbox supports numerous normalization modes, for a detailed overview we refer to [2].

Figure 12 compares the two most crucial normalization modes: the block-wise (default) normalization modes, and nonlinear detection mode. Authors recommend checking always the period-wise (block-wise) noise and nonlinearity levels first because using many realizations and simultaneously showing the improved nonlinearity level (i.e. experiment-wise normalization) might give a wrong message to the user since it converges to zero. The second most useful normalization mode is the nonlinear detection mode which can be used to detect the presence (significance) of nonlinearities (nonlinear variability mode). The transition between different noise and nonlinear quantities can be seen in Figure 12as well.

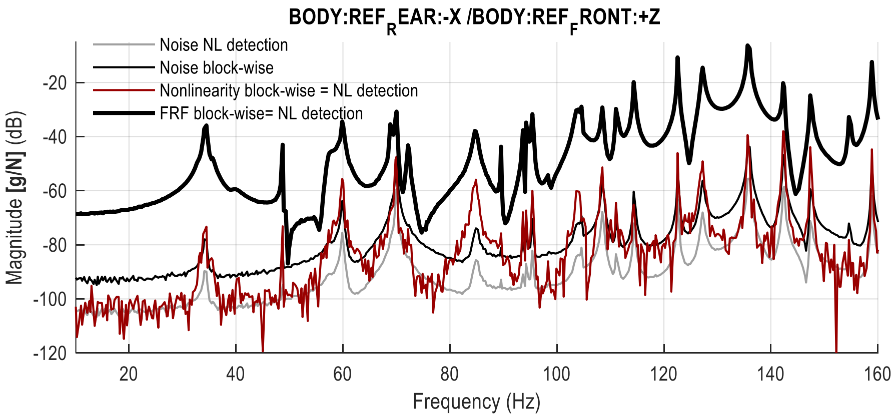

Figure 12: FRFs, noise and nonlinearity estimates shown at different normalization modes at high level of excitation. Block-wise mode refers to the period-wise normalization of noise and nonlinear distortions estimates. Nonlinear (variability) detection mode is the mode where the nonlinearities are shown w.r.t one block, noise is shown wr.t. the all blocks. Observe that the BLA FRF G_{BLA} and the nonlinear estimates (G_S) remains the same, only the noise estimates have been moved from the block-wise normalization (G_E) to the measurement-wise normalization ($\hat{\sigma}^2_{\hat{G}}$).

6.5.3 Using only one block

Authors recommend using as many realizations (and periods) as possible. However, it is sometimes not possible to measure long. Using the proposed multisines, it is possible to have a rough estimate about the nonlinearity and noise levels using at least 1 realization and 2 periods. In the classical literature [8] the 1 realization case is called fast method, the multiple realizations case (with full multisines) is called robust method, where robust refers to the robustness estimate of the nonlinear and noise quantities.

Figure 13 shows another interesting case to highlight the capabilities of the toolbox. A car frame is a lowly damped structure. It has been tested using 100 blocks of MIMO multisines, each block is 10,2 seconds long, the transient is 20.4 seconds (i.e. 2 blocks) long. The figure illustrates the situation what happens if only one measured block is available: the classical H1, windowed H1 [17], and the automated choice (in this case a special implementation of Local Rational Method [12]) are compared with each other. As can be seen, the automated choice has the best performance.

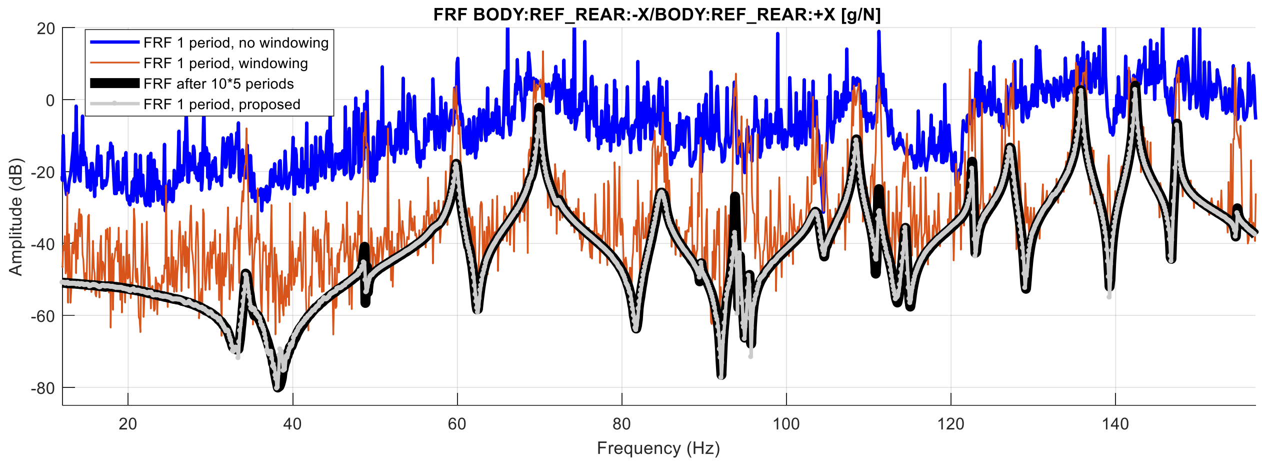

Figure 13: An FRF estimate of a lowly damped vehicle. The black line shows the H1 estimate using fifty periods. The blue line shows the classical H1 estimate using one period, the red line shows the H1 estimate with Hanning windowing using one period. The gray line shows the LRM estimate using one period.

7 Conclusions

In this work a toolbox has been introduces based on the MIMO Best Linear Approximation framework to provide a user-friendly interpretation of the nonlinear behavior of measurement data by extracting user relevant information. The proposed toolbox turned out to be useful for modelling FRFs because:

- it requires minimal user-interaction, and an expert user is not needed.
- orthogonal excitation signals have been provided to optimally excite structures with multiple inputs,
- the reference, input and output measurements were nonparametrically characterized,
- advanced frequency response matrix estimation is provided

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