Frequency Response Function Estimation for Systems with Multiple Inputs using Short Measurement: A Benchmark Study

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ABSTRACT
The aim of the paper is to introduce an identification method for industrial vibro-acoustic systems with multiple inputs using (very) short measurement. The classical time-consuming phase resonance (or normal modes) testing procedures are nowadays almost fully substituted by frequency response functions (FRFs) methods which are used to obtain parametric models (e.g. resonance frequencies, modes shapes in modal analysis, or e.g. state-space models in control engineering).

This paper presents a novel nonparametric FRF estimation methodology which allows the user to efficiently estimate broadband transfer functions of multiple input systems using only one block of measurement (disturbed by transient term and noise). The proposed method is a novel extension of already existing local parametric techniques used for SISO identification. In order to assess the performance of the proposed method, a benchmark study is performed on a tire-suspension measurement where the candidate estimator performance is compared to classical H1 and to the windowed-overlapped H1 techniques.

Keywords: MIMO systems, nonparametric estimation, short measurement analysis, system identification

1. INTRODUCTION

The goal of this paper is to introduce a simple but efficient state-of-the-art method for nonparametric modeling vibrating structures using (very) short measurement in real-life experimental situations.

Vibro-acoustic measurements are the results of vibration testing (structural dynamic testing) techniques. Vibration testing takes place (typically) at the end of the development process and it requires a physical prototype. There is a very high time pressure due to limited availability of the fully assembled physical prototypes, and due to the tight deadline of the production date. On the other hand, the vibration testing methods are very important because they help to improve the product quality and to avoid safety and comfort issues. The goal of the vibration testing is to obtain experimental data of the whole vibrating structure such as road and air vehicles. Using these data, it is possible to validate and improve the dynamic models of systems under test.

The increasing needs for higher accuracy and faster testing techniques inspired a lot of international researchers [1] [2] [3]. As a result, time-consuming testing procedures, for instance, phase resonance method ([4] [5] [6]) are nowadays almost fully substituted by so-called phase separation techniques that find the modes by evaluating the broadband frequency response functions (FRFs)

When considering a MIMO (multiple input, multiple output) measurement setup, it seems to be handy to excite each of the inputs separately. This technique raises some important questions. First, in this case, significantly more experiments are needed which drastically increases the time needed to measure and analyse the FRFs. Second, the principle of superposition is only valid for linear systems. Therefore, in practice, all input channels are simultaneously excited.

According to the traditional signal processing methodology, for systems with N_i input at least N_i uncorrelated segments of the excitation is needed, otherwise the set of linear equations used to estimate FRFs will suffer from rank deficiency- due to high degrees of freedom. Further, in order to reduce the effect of disturbing noise, multiple repetitions of each N_i uncorrelated segments are needed. These repetitions of blocks are also important for the transient elimination because for most of the FRF estimation methods it is crucial that the steady-state signals are considered, otherwise the transient term is included in the estimated FRF model, and therefore, it will not be an adequate representation of the system. When many blocks are available, it is the best practice to discard the first few blocks (the transient periods or in other words, the delay blocks). When it is not possible, different windowing and/or overlapping methods are used in practice.

This paper presents a novel nonparametric estimation methodology which allows the user to efficiently estimate broadband FRFs of multiple input systems using only one realization/one block of measurement (disturbed by transient term and noise). The numerical results are obtained by the use of self-developed toolbox called SAMI (Simplified Analysis of Multiple Inputs) [7] [8]. In order to assess the performance of the proposed method, a benchmark study is performed on a tire-suspension measurement where the candidate estimator performance is compared to classical H1 and to the windowed-overlapped H1 techniques.

This paper is organized as follows. Section 2 briefly describes the considered systems and the main assumptions applied in this work. Section 3 discusses the nonparametric estimation technique. In Section 4 the description and analysis of the experiment are given. Conclusions can be found in Section 6.

2. BASICS

Considered systems

The dynamics of a linear MIMO system can be nonparametrically characterised in the frequency domain by its Frequency Response Matrix (FRM) [9] G at discrete frequency index k, which relates the inputs U to outputs Y as follows:

$$Y(k) = G(k)U(k) \tag{1}$$

In this work arbitrary an number of input and output channels are considered and the underlying systems are BIBO stable physical systems [10]. For the sake of simplicity, the frequency indices will be omitted, and it is assumed to understand each quantity at frequency index k.

This system represented by the G is linear when the superposition principle is satisfied in steady-state, i.e.:

$$Y = G(a+b)U \ = a\ GU + b\ GU = (a+b)\ GU \tag{2}$$

where a and b are scalar values. If G is constant, for any a, b (and excitation), then the system is called linear-time invariant (LTI). On the other hand, when G varies with a and b (and the variation depends also on the excitation signal – e.g. level of excitation, distribution, etc.) then the system is called nonlinear.

Because time-varying systems are often misinterpreted as nonlinear systems, it is important to mention that when G varies over the measurement time, but at each time instant the principle of superposition is satisfied, then the system is called linear time-varying (LTV) [11].

In this work we consider linear time-invariant stable (damped) mechanical (vibrating) systems. The proposed framework makes use of the classical instrumentation and measurement setups [24].

MIMO baseline model

For the sake of simplicity – without loss of generality – we will focus on systems with three inputs and two outputs, a block illustration of such a setup is shown in Figure 1. This allows to illustrate the additional problems that appear when moving from SISO to MIMO for a minimal increase of the complexity.

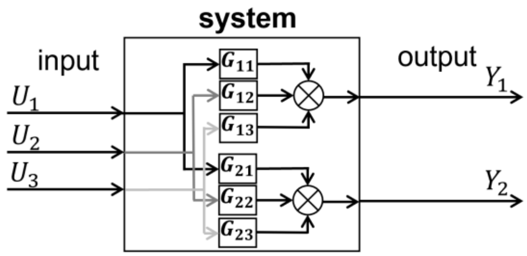

Figure 1. Illustration of a system with three inputs and two outputs.

The straightforward extension of the ideal SISO FRF case can be formulated in the frequency domain at the excited frequency bins (lines) as follows:

$$Y = GU \iff \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} \mathrm{G}_{11} & \mathrm{G}_{12} & \mathrm{G}_{13} \\ \mathrm{G}_{21} & \mathrm{G}_{22} & \mathrm{G}_{23} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} \tag{3}$$

where the indices of the input and output data refer to channel number.

Even when there are multiple periods of excitation applied, the above-mentioned set of linear equations suffers from the degrees of freedom: there are 6 unknown parameters and only 2 independent equations. To solve this issue, it is needed to increase the number of experiments. In case of 3 inputs there are at least 3 experiments needed such that the new base-line equation is defined as follows:

$$\begin{bmatrix} Y_{[1,1]} & Y_{[1,2]} & Y_{[1,3]} \\ Y_{[2,1]} & Y_{[2,2]} & Y_{[2,3]} \end{bmatrix} = \begin{bmatrix} \mathrm{G}_{11} & \mathrm{G}_{12} & \mathrm{G}_{13} \\ \mathrm{G}_{21} & \mathrm{G}_{22} & \mathrm{G}_{23} \end{bmatrix} \begin{bmatrix} U_{[1,1]} & U_{[1,2]} & U_{[1,3]} \\ U_{[2,1]} & U_{[2,2]} & U_{[2,3]} \\ U_{[3,1]} & U_{[3,2]} & U_{[3,3]} \end{bmatrix} \tag{4}$$

where the first digit of the indices in square brackets refers to channel and the second digit refers to the number of experiment. To obtain a classical FRM estimate, at each excited frequency index (4) must be solved.

Hadamard decorrelation technique

The solvability of the above-mentioned linear algebraic equation strongly depends on the condition number (i.e. the randomness) of the excitation signal matrix U. In classical MIMO identification, one of the most often applied solution to this problem is the use of Hadamard decorrelation technique (known as +- technique as well [12]: a square matrix (whose entries are either +1 or −1) is elementwise multiplied with one single realization of the signal. For instance, in case of 3x3 situation, one has to take 4x4 Hadamard matrix, and use the upper left 3x3 elements so that the excitation signal U becomes:

$$U_{Hadamard} = \begin{bmatrix} U_1 & U_1 & U_1 \\ U_1 & -U_1 & U_1 \\ U_1 & U_1 & -U_1 \end{bmatrix} \tag{5}$$

Classical H1 estimator

The classical spectral density based H1 estimator assumes ideal input measurement, or in other words, no noise on the input signal [13]:

$$\hat{G} = S_{YU} S_{UU}^{-1} \tag{6}$$

where S_{UU}, S_{YU} is the autoinput and cross-power spectra, and x^{-1} is the inverse of x.

In practice, the quantities in (6) should be estimated. The inversion of the input signal matrix U also implies that the input signals should be uncorrelated. In the traditional case, when random noise excitation is considered, different methods are suggested to be used to estimate the power spectra. The traditional methods include, for instance, windowing techniques, data overlapping, correlation-based techniques.

3. PROPOSED METHODOLOGY

A realistic baseline model

Equation (3) and Figure 1 show an idealistic steady-state scenario without measurement noise. Considering a realistic model (3) results in the following model:

$$Y_{measured} = Y + E + T = GU + E + T \tag{7}$$

where E is the measurement noise – assumed to be i.i.d. white Gaussian noise, and T represents the transient term.

Periodic excitation helps to overcome possible issues with leakage, helps to reduce the noise level, and, when the measurement is sufficiently long, transient contaminated periods can be simply discarded. In many cases, due to the very low damping of the underlying system or insufficient number of periods this discarding technique can become unpractical. In '60s windowing methods were developed to suppress the errors of the leakage and transient. However, nowadays the computational power is significantly higher which opened the possibility to use solutions with a better leakage rejection at a cost of making more calculations.

Because the FRF and the transient are smooth it is possible to use polynomial and smooth basis function approximations such as the Local Polynomial Method (LPM) [9] or the flexible B-splines [14] [15]. The drawback is that these techniques have issues with highly damped systems such that the peaks in the FRFs are usually underestimated. To improve the estimates at a cost of more parameters to be estimated, the Local Rational Method (LRM) has been suggested [9]. The LRM still uses the property of the smoothness of the FRF and transient transfer function, but also uses the fact that they inherently have common denominators. This paper considers a frequency-wise, iteration free, direct implementation of the LRM. In the process, a sliding processing window is used. Within this window a polynomial function of degree of d is estimated around the center point called central frequency (frequency index k). Around this central frequency line, the narrow band of $\pm d/2$ radius is given by:

$$r = -round\ down\left(\frac{d}{2}\right) \ldots 0 \ldots round\ up\left(\frac{d}{2}\right) \tag{8}$$

The measured output at the excited frequency index k in a radius of r is given by:

$$Y_{measured}[k+r] = \mathrm{G}[k+r]U[k+r] + T[k+r] + E[k+r] = \frac{G_p}{D}U + \frac{T_p}{D} + E[k+r] \tag{9}$$

where G_p, T_p, D are polynomials of order d, k is the so-called central frequency, and r is a narrow band.

The polynomials are given by:

$$G_p = \sum_{s=0}^{d} g_s r^s = \overbrace{g_0}^{G} + g_1 r + g_2 r^2 + \cdots + g_d r^d \tag{10}$$

$$T_p = \sum_{s=0}^{d} b_s r^s = \overbrace{t_0}^{G_T} + t_1 r + t_2 r^2 + \cdots + t_d r^d \tag{11}$$

$$D = 1 + \sum_{s=1}^{R} d_s r^s = 1 + d_1 r + d_2 r^2 + \cdots + d_d r^d \tag{12}$$

The cost function

Multiplying (9) with D makes the identification problem linear-in-parameters. The solution is given by:

$$\hat{\theta}_k \triangleq arg \min_{\theta_k} \sum_r \left|Y[k+r]D - G_p U[k+r] - T_p\right|^2 \tag{13}$$

where $\hat{\theta}_k$ is the parameter estimate of the LRM model (i.e. the polynomials) at frequency bin k.

Per output, for each frequency line there are $N_i(d+1) + d$ unknown parameters to be estimated ($N_i(d+1)$ in G_p, $d+1$ in T_p, d in D). This means, for the algorithm to work, in the processing window should be at least so many datapoints.

4. EXPERIMENTAL ILLUSTRATION

Short description of the tire suspension measurement

This section concerns the industrial vibration testing of a tire-suspension system, for an illustration see Figure 2. This test setup was used to validate the component-based TPA (transfer path analysis) methodology [16]. As part of this methodology, accurate FRF measurements are required. The tire suspension is excited at five different points. The excitation signal is pseudo-random noise (multisines). The sampling frequency is 1600 Hz, the period length 2048 resulting in a frequency resolution of 1.28 Hz. There are in total 11 independent realizations, 22 repeated blocks (periods) per realization. The range of excitation is between 20 and 600 Hz. The input (force) and output (accelerations) signals are measured.

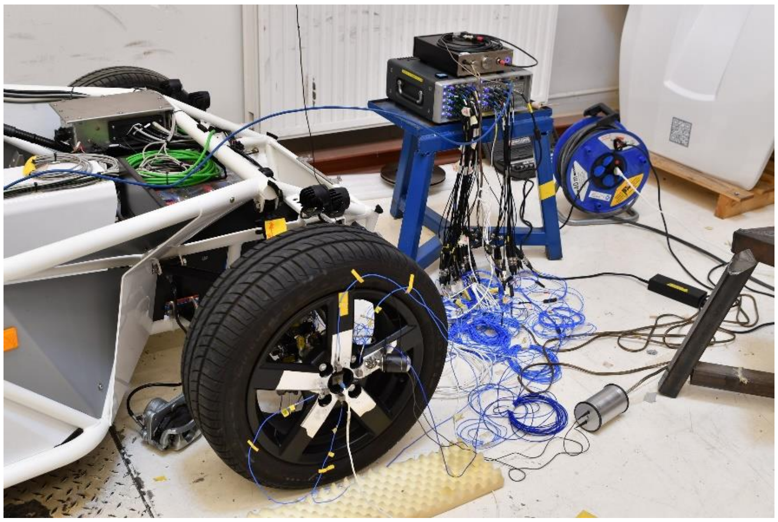

Figure 2: Illustration of an experimental test setup used for component-based testing

Data pre-processing

The data processing is fully automated by the SAMI toolbox: segmentation of data, trends (such as the mean values) from the individual segments (blocks) are removed [17] [18], the transient is analysed. Figure 3 shows the visualization of toolbox transient check-up routine. The left side of the figure shows the acceleration (output) measurement at one of the important channels. In order to determine the length of the transient (i.e. the number of delay blocks), the last block (period) – assumed to be nearly in steady-state – is subtracted from every preceding block. Because the transient decays as an exponential function, the differences are shown in logarithmic scale [18].

The five input (force) signals and their noise estimates are shown in frequency domain in Figure 4. The measurement has an acceptable quality (SNR is around 40 dB) suitable to fulfil the assumptions needed for H1 estimator.

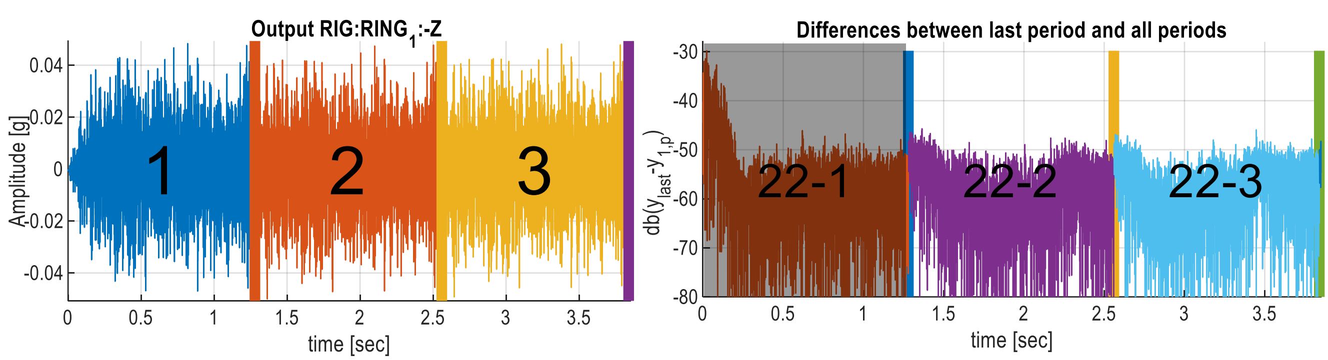

Figure 3: Periodicity at the one of the output channels. The left figure shows the block repetitions for the first 3 blocks (accelerometer data). The right figure shows the differences between the last block (22) minus the first three blocks in dB scale. Observe in the second figure the fast decay, this is the transient. The greyed area refers to the automatically detected transient (delay) block.

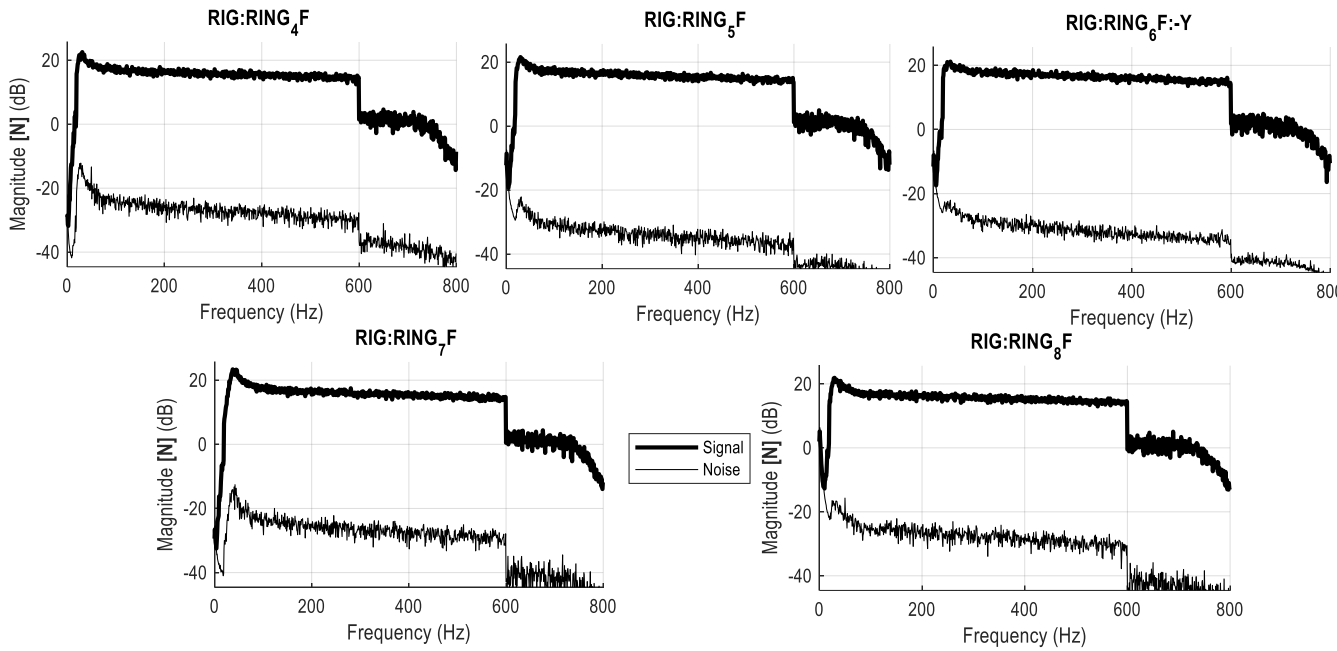

Figure 4: The five measured input (force) signals are shown. Thick lines refer to the signals. Thin lines refer to noise estimates.

Testing process

In the testing process the results of the classical nonparametric FRF estimator are compared to the results of the proposed method. In case of the LRM method, second order polynomials are considered. For 5 inputs and second order polynomials should be at least 17 datapoints in the processing window. In the test case, a bandwidth of 20 is used because this order provides a good balance between sharpness (accuracy) of estimate and noise elimination. As for the classical estimators, two cases are considered. First one is the unwindowed H1 estimator. The second one is the Hanning windowed H1 estimator with 67% of overlap.

The test results are given in benchmarking style. In the first run, zero delay (transient) block has been used with 1, 3, 5, and 10 independent realizations. For each realization there are 1, 2 and 10 repeated blocks available. In the second run, one delay block has been used (discarded). In order to compare the performances of the estimators, in this work a cross-validation technique is used on an independent dataset (the last realization). First, the relative root mean square (rms) error is calculated in the time domain as:

$$rmse = rms(\hat{y} - y_{measured})/rms(y_{measured}) \tag{14}$$

where $\hat{y}$ is the modelled output signal, $y_{measured}$ is the measured output signal. Then the goodness of fit is calculated as:

$$goodness\ of\ fit = (1 - rmse) * 100\ [\%] \tag{15}$$

100% of goodness of fit means that the whole data is modelled without error, 0 or lower goodness of fit means that the output produced by the model is entirely inaccurate.

Table 1 shows the overview of the cross-validation results. Here, four scenarios are studied with the windowed H1 and LRM method. First, Figure 5 shows the estimation results using only one transient disturbed realization-block (i.e. very first data segment). Observe that the quality of H1 estimator is highly insufficient whereas LRM already provides acceptable results. Figure 6 shows the scenario with 5 independent blocks – what is theoretically needed to solve the classical MIMO FRF problem. The classical H1 estimation quality is still insufficient for most applications. Figure 7 shows the scenario with 10 independent blocks. The H1 results are improved, but still it is very noise. Figure 8 shows the results of 10 realization and 10 repeated transient free blocks. Observe that even with large number of parameters, the windowed H1 estimate can provide very noisy estimates (see the last transfer function).

Overall conclusion is that the LRM outperforms the classical approaches. In the worst-case situation, when only one block is used, the goodness of fit of LRM is 94.4%. In the best-case scenario, when all data records are used, 99.9% is the goodness of fit. Among H1 estimators, the best overall performing is the windowed, overlapped H1 with the best possible fit of 98,2% and the worst fit -1546,6%. In the idealistic scenario, when long, persistently exciting data records are available, the classical H1 delivers better result than the windowed-overlapped H1 version. In both H1 cases, it is difficult to predict the goodness of fit as it is not as systematic as the LRM estimation.

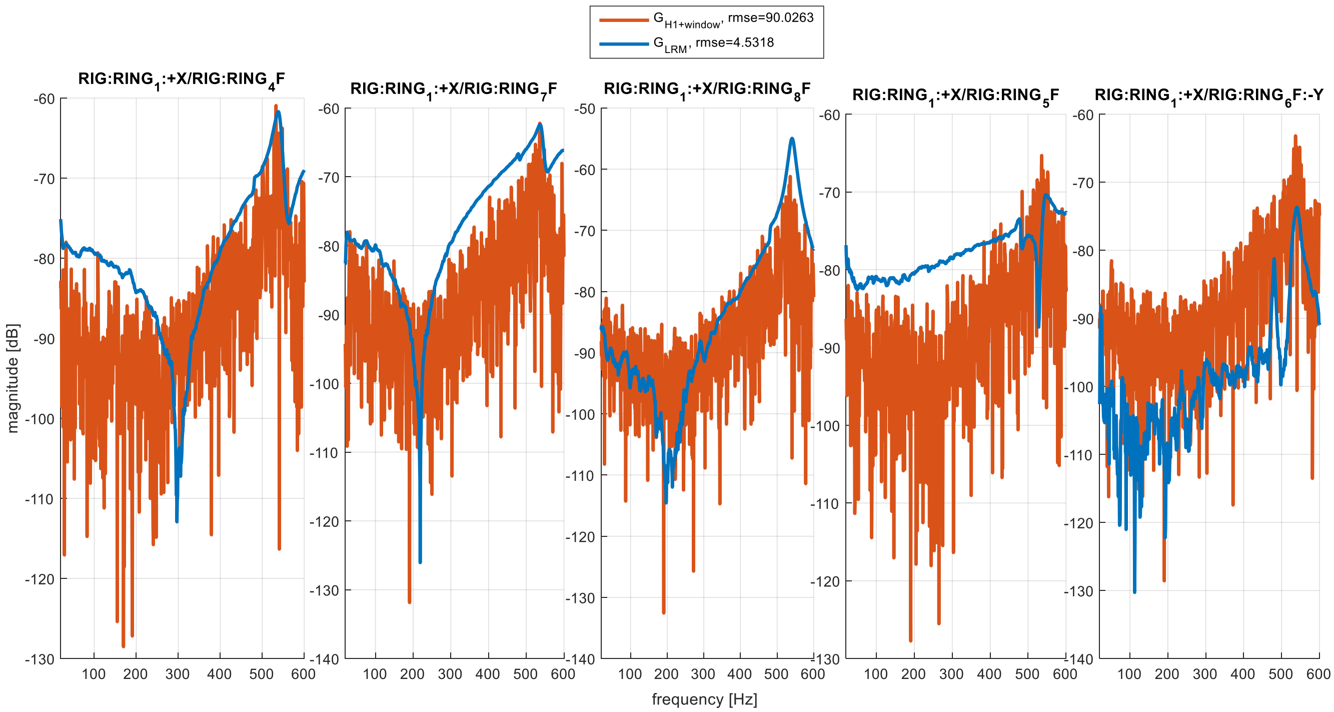

Figure 5: H1 Hanning windowed estimate and LRM estimates are shown for the case using 0 delay block, 1 realization, 1 block. RMSE refers to the relative mean squared error calculated on an independent segment of data.

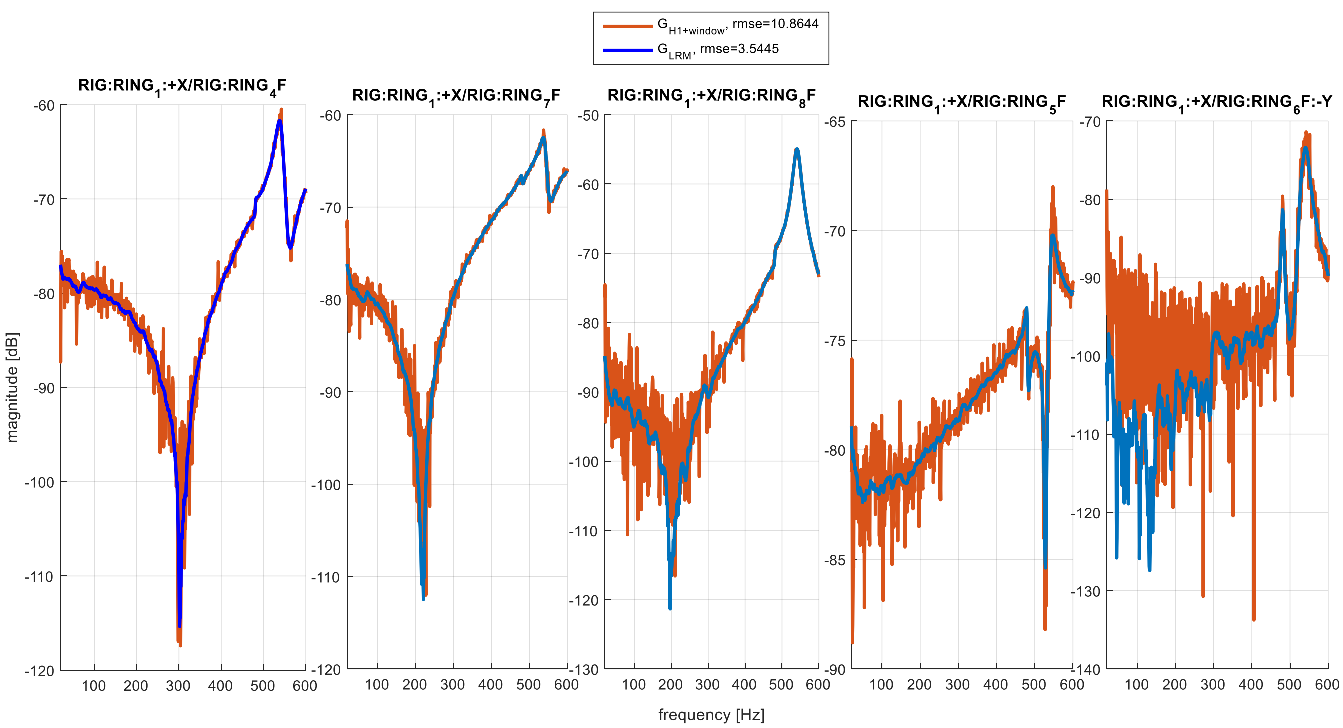

Figure 6: H1 Hanning windowed estimate and LRM estimates are shown for the case using 0 delay block, 5 realization, 1 block. RMSE refers to the relative mean squared error calculated on an independent segment of data.

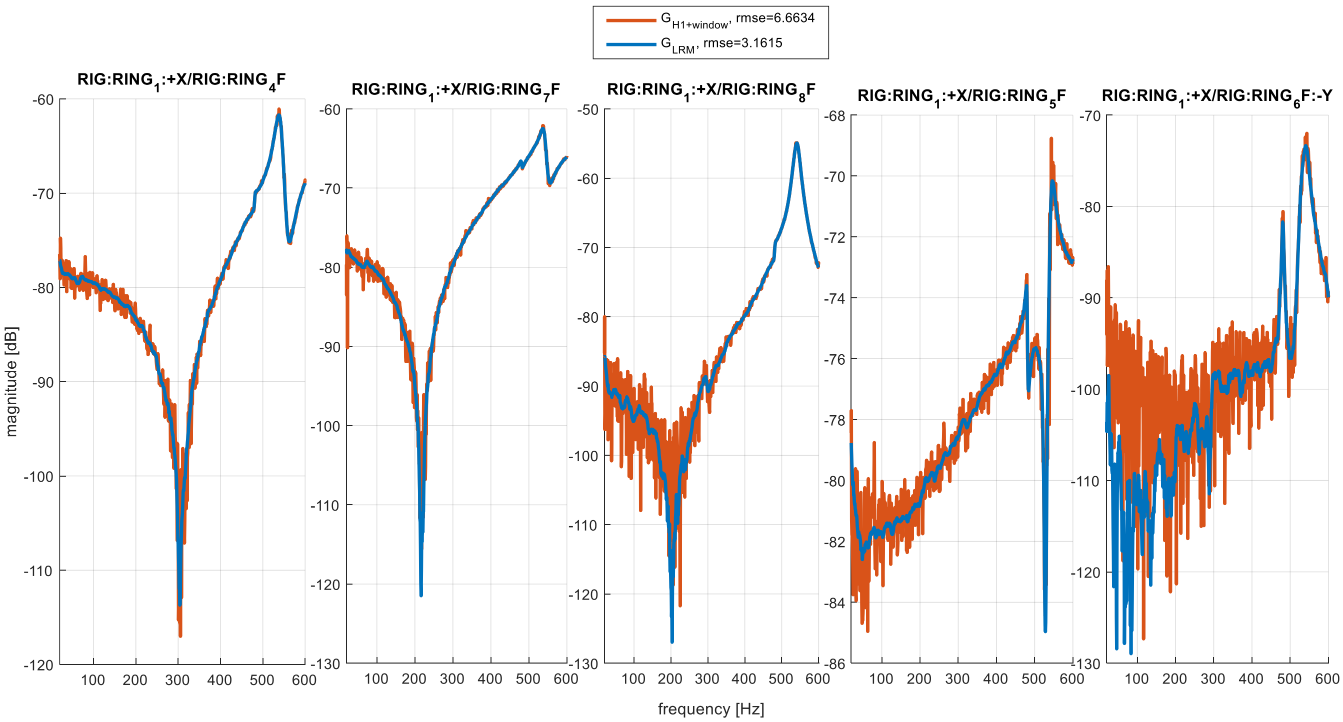

Figure 7: H1 Hanning windowed estimate and LRM estimates are shown for the case using 0 delay block, 10 realization, 1 block. RMSE refers to the relative mean squared error calculated on an independent segment of data.

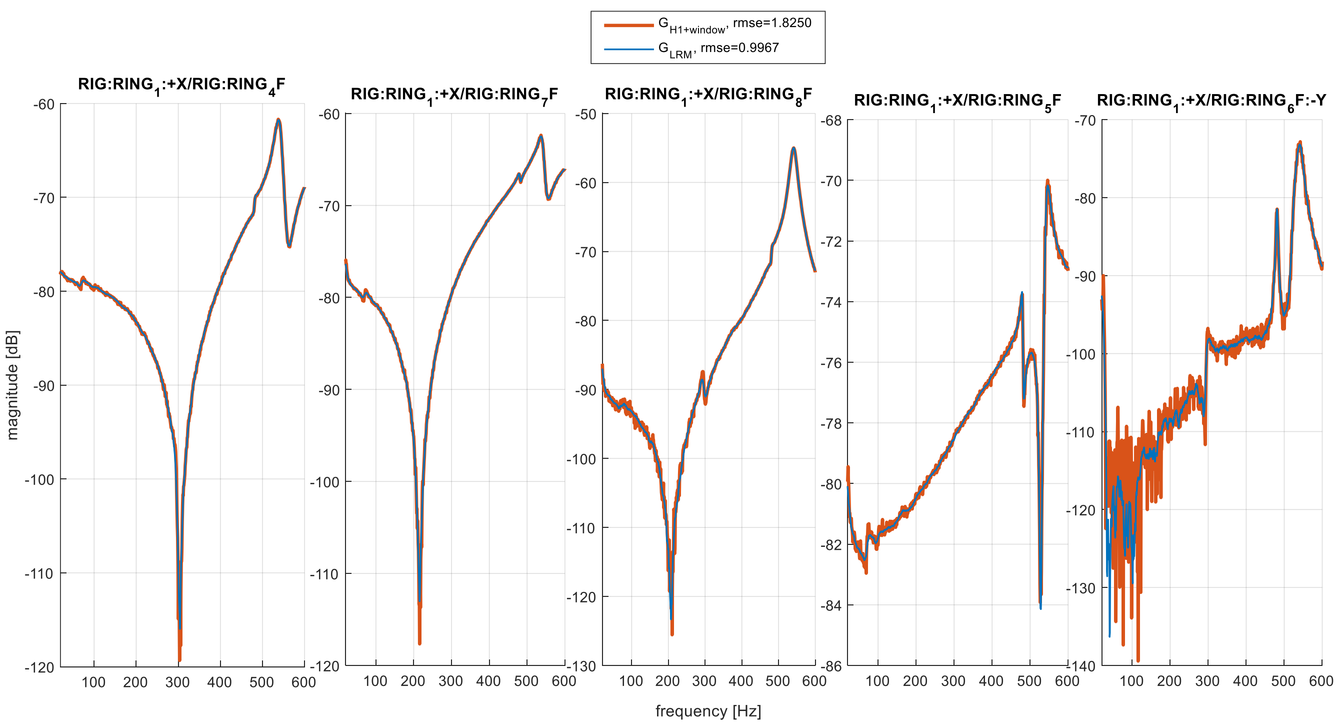

Figure 8: H1 Hanning windowed estimate and LRM estimates are shown for the case using 1 delay block, 10 realizations, 10 blocks. RMSE refers to the relative mean squared error calculated on an independent segment of data.

Table 1. Overview of the cross-validation results. Green background indicates the best estimation result, orange background indicates the worst estimation result for a particular case.

Figure number	Delay blocks	Number of blocks	Realizations	Goodness of fit [%]		
				H1	windowed H1	LRM
5	0	1	1	6,7	10,0	94,4
	0	2	1	19,4	-189,9	95,8
	0	10	1	-2371,7	-166,1	96,4
	0	1	3	35,7	89,1	96,2
	0	2	3	58,4	93,5	96,7
	0	10	3	63	94,9	97,0
6	0	1	5	59,9	93,3	96,7
	0	2	5	86,7	95,6	97,1
	0	10	5	92,9	96,5	97,3
7	0	1	10	93,0	96,4	97,7
	0	2	10	96,1	97,2	97,8
	0	10	10	98,4	98,2	98,4
	1	1	1	6,1	9,9	95,5
	1	2	1	-843,9	-1546,6	96,0
	1	10	1	-3215,6	-249,8	96,5
	1	1	3	37,4	88,3	96,6
	1	2	3	-3006,4	93,6	96,8
	1	10	3	-367,1	95	97,0
	1	1	5	70,1	90,5	97,0
	1	2	5	80,4	94,3	97,1
	1	10	5	90,6	96,5	97,3
	1	1	10	96,7	93,9	97,7
	1	2	10	97,6	96,6	97,8
8	1	10	10	98,5	98,2	99,0

5. CONCLUSIONS

In this work a novel, nonparametric FRF estimation approach was developed to provide accurate results in case of short available data. The proposed method outperformed the classical H1 and windowed-overlapped H1 approaches.

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